Fractional Set Theory: A System for Microtonal Music Analysis

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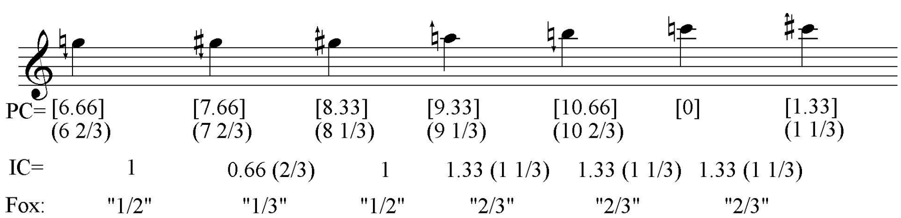
In **fractional set theory**, microtones are represented as pitch classes in a decimal format. For example, the quartertone between E [4] and E♭ [3] is [3.5], as seen in violin IIa.

Ives, Charles. Symphony No. 4, movement IV. m. 32

Christopher Fox uses sixth-tones in *skin*. A selection from his pitch chart in *Contemporary Music Review* is modified here into FST format. In *skin*, all ICs other than a semitone are either sixth- or third-tones (IC1⁄3 or IC2⁄3, respectively), PCs and ICs may be represented as fractions instead of decimals.



In his 2005 *PNM* article, **Jason Eckardt** uses mod-24 set theory to represent quartertones in his piece *Polarities*. Simply divide the mod-24 PCs by 2 to get the FST PC. Example 7C of his article shows PC set [0, 1, 2, 7, 8, 12], which translates to [0, 0.5, 1, 3.5, 4, 6]. For an example of 19TET with mod-19 set theory, see **Hubert Howe's** 1993 article "19- Tone Theory and Applications" located at http://qcpages.qc.edu/~howe/articles/19-Tone%20Theory.html

Pitch Class Sets, Interval Classes, Interval Class Vectors

The 7TET scale [0, 1.71, 3.43, 5.14, 6.86, 8.57, 10.29] contains only three unique intervals: IC 1.71 (a second), IC 3.43 (a third) and IC 5.14 (a fourth)

The 12TET scale contains six unique intervals, therefore its interval class vector has six columns. E.g., [0146] has an ICV of <111111> (it contains one of each interval type).

The ICV of the 7TET scale, therefore, has only three columns. E.g., [0, 3.43, 6.86] has an ICV of <021> (it contains no seconds, two thirds, and one fourth)

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Gerard Grisey's *Prologue* for viola (1978) uses a notation that includes quartertones and smaller deviations marked with an arrow (up or down). This sequential figure (after the first fermata in the second system) treats these smaller deviations as eighth-tones.

PC [4, 9.5, 11] [3.75, 9.25, 10.75] [3.5, 9, 10.5] [3.25, 8.75, 10.25] [3, 8.5, 10]
PC set [4, 9.5, 11] is sequenced down by IC 0.25 until it reaches [3, 8.5, 10].

Kyle Gann's *Triskaidekaphonia* (2005) uses a 29-note scale derived from just intonation whole number ratios 1 to 13, with PC 2 as fundamental. For example, 13:12 generates a pitch 138.6 cents higher than tonic (E♭+38.6 cents). To get the PC in FST, divide the cents (found in Gann's program notes) by 100, and add 2. Using this method, 13:12 yields PC 3.386. The scale for *Triskaidekaphonia* in FST is PC set [0.176, 0.494, 0.717, 2, 3.386, 3.506, 3.65, 3.824, 4.039, 4.312, 4.669, 4.892, 5.156, 5.474, 5.863, 6.351, 6.542, 6.98, 7.513, 7.825, 8.175, 8.366, 9.02, 9.825, 10.137, 10.405, 10.8844, 11.331, 11.688].

Nolan Stolz's *What The Waves Tell Me* for cello (2009) is entirely in just intonation, created by natural harmonics of the strings, which are tuned to the harmonics of the fundamental, PC 11.31. String IV is tuned to the 2nd partial PC 11.31, III is tuned to the 3rd partial PC 6.33, II to the 5th partial PC 3.17, and I is tuned to the 7th partial PC 9. Only the first 7 harmonics are used on any given string, so the PC sets are: String I [0.86, 4.02, 6.69, 9]; String II [0.86, 3.17, 7.03, 10.19]; String III [1.35, 4.02, 6.33, 10.19]; String IV [3.17, 6.33, 9, 11.31] [0.86, 1.35, 3.17, 4.02, 6.33, 6.69, 7.03, 9, 10.19, 11.31] Ordering the PCs reveals some interesting ICs such as IC(0.49) derived from PCs (0.86, 1.35), IC(0.33) derived from PCs (6.69, 7.02), and IC(0.36) derived from PCs (6.33, 6.69).

Enharmonics and Binomial Representation

Enharmonics can be troublesome in microtonal music notation and when naming notes. "Binomial representation" allows for greater specificity. **Pitch classes** can have multiple names. For example, PC 0.5 can be called either "C 1⁄4 sharp" or "D 3⁄4 flat." The former is represented as PC<0.5, 2>. The ***pitch class*** is on the left, and the ***name class*** (mod-7) is on the right (A=0, B=1, C=2, and so on). PC<0.5, 2> ("C 1⁄4 sharp") can be enharmonically respelled as PC<0.5, 3>, or "D 3⁄4 flat."

"**Binomial representation**" may also be applied to **interval classes**. For example, a minor second raised by a quartertone (a "neutral second") is represented as IC<1.5, 2> (e.g., F to G-quarter-flat). The ***interval class*** is on the left, and the ***interval name class*** (mod-*n*) is on the right (unison=1, second=2, third=3, etc.). This interval can be enharmonically respelled as an augmented unison (raised by a quartertone) IC<1.5, 1> (e.g., F to F-three-quarters-sharp). IC<4.5, 4> is a perfect fourth lowered by a quartertone (a "minor fourth") IC<4.5, 3> is a major third raised by a quartertone.

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