

Fractional Set Theory: A System for the Analysis of Microtonal Music

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In **fractional set theory**, microtones are represented as **pitch classes** in a decimal format. For example, the quartertone between E (4) and E ♭ (3) is 3.5, and the quartertone between B(11) and B ♭ (10) is B quarterflat 3.5

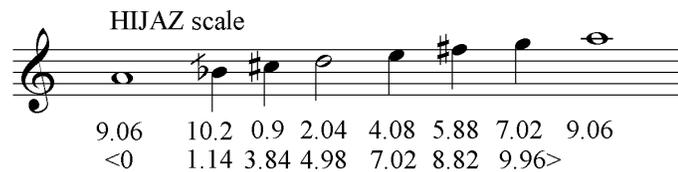
Rast



(scale given in Powers 2005)

Fractional Set Theory can offer a greater exactitude in pitch representation. The hijaz (*hecaz*) scale in Turkish art music is shown here in Ezri-Arel notation (from Signell 2008) with **fractional pitch classes** listed below. The PC set is also given with tonic designated as “0.”

HIJAZ scale



9.06 10.2 0.9 2.04 4.08 5.88 7.02 9.06
<0 1.14 3.84 4.98 7.02 8.82 9.96>

Interval classes are also represented in a decimal format. The interval class from $\wedge 2$ to $\wedge 3$ in the Turkish hijaz scale shown above is 2.7, or 270 cents (PC 10.2 up to PC 0.9).

The 7TET scale [0, 1.71, 3.43, 5.14, 6.86, 8.57, 10.29] contains only three unique intervals: IC 1.71 (a second), IC 3.43 (a third) and IC 5.14 (a fourth). The tuning of the mbira dza vadzimu according to Tracey 1970 is <0, 1.62, 3.43, 5.16, 6.84, 8.52, 10.3>, which is very close to 7TET.

Interval class vectors

The 12TET scale contains six unique intervals, therefore its interval class vector has six columns. E.g., [0146] has an ICV of <111111> (it contains one of each interval type).

The ICV of the 7TET scale, therefore, has only three columns. E.g., [0, 3.43, 6.86] has an ICV of <021> (it contains no seconds, two thirds, and one fourth)

The “Zalzal” scale (Schulter 2003)/Nerz Rast (Powers 2005) is PC set <0, 2, 3.5, 5, 7, 8.5, 10>. The intervals of the adjacent scale steps are: 2, 1.5, 1.5, 2, 1.5, 1.5 and 2 (all major seconds and neutral seconds). The scale is quartertonal, so the ICV will have 12 columns. The ICV for the Nerz Rast/Zalzal scale is <004302410520>. Notice that the interval class with the highest cardinality is the perfect fourth (IC 5) with 5 instances: C to F, D- G, E-quarter-flat to A quarter-flat, F to B-flat, and G up to C. Also notice the first tetrachord (C-D-Equarterflat-F) when transposed up IC 5, completes the scale: F-G-Aquarterflat-Bb.
(given on page 2 of handout)

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overtone ratios:

	1/1	9/8	27/22	4/3	3/2	18/11	16/9	2/1
PC:	0	2.04	3.55	4.98	7.02	8.53	9.96	0
simplified								
PC:	0	2	3.5	5	7	8.5	10	0

Zalzal scale/ Nerz Rast

Fractional Harmonics (including “half harmonics”)

4	5	5.5	6	7	
		-14	-49	+2	-31

If PCs 0, 3.86, 7.02 and 9.69 are presented within one octave, then they may be labeled as harmonics 4 : 5 : 6 : 7 of a fundamental two octaves below (PC 0). If PC 5.51 is introduced within the same octave, it would be the 11th harmonic of the fundamental an octave below the former. $\langle 0, 3.86, 5.51, 7.02, 9.69 \rangle$ may be labeled as harmonics 8 : 10 : 11 : 12 : 14 or as 4 : 5 : 5.5 : 6 : 7.

The “Zalzal” scale may be labeled as harmonics 1 : 1.125 : 1.272 : 1.333 : 1.5 : 1.636 : 1.778 (see the overtone ratios given above the scale). To calculate the fractional harmonics, divide the higher number of the ratio by the lower number.

	+12	-40	+20	+45	-12	+6	-40

Central Javanese saron (in slendro tuning) from the Kyai Rengga Manis Everist gamelan at the National Music Museum (Vermillion, SD)
 With ICs rounded to the nearest eight-tone, the PC set is $\langle 0, 2.5, 4.75, 7.25, 9.5 \rangle$ (with PC 0.6 designated as “0”). This is very close to 5TET: [0, 2.4, 4.8, 7.2, 9.6].

Enharmonics and Binomial Representation

Enharmonics can be troublesome in microtonal music notation and when naming notes. “Binomial representation” allows for greater specificity. **Pitch classes** can have multiple names. For example, PC 0.5 can be called either “C 1/4 sharp” or “D 3/4 flat.” The former is represented as PC $\langle 0.5, 2 \rangle$. The *pitch class* is on the left, and the *name class* (mod-7) is on the right (A=0, B=1, C=2, and so on). PC $\langle 0.5, 2 \rangle$ (“C 1/4 sharp”) can be enharmonically respelled as PC $\langle 0.5, 3 \rangle$, or “D 3/4 flat.”

“**Binomial representation**” may also be applied to **interval classes**. For example, a minor second raised by a quartertone (a “neutral second”) is represented as IC $\langle 1.5, 2 \rangle$ (e.g., F to G-quarter-flat). The *interval class* is on the left, and the *interval name class* (mod-*n*) is on the right (unison=1, second=2, third=3, etc.). This interval can be enharmonically respelled as an augmented unison (raised by a quartertone) IC $\langle 1.5, 1 \rangle$ (e.g., F to F-three-quarters-sharp).

A bibliography of the works cited in the paper is available upon request.

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